

Contents

Introduction

The Pattern of Our Number System

Chapter 1 - First Wave	1
Chapter 2 - Second Wave	35
Chapter 3 - Counting is Adding	49
Chapter 4 - Even and Odd	53
Chapter 5 - Doubles and Halves	59
Chapter 6 - Breaking up Single-Digit Numbers into the Sum of Two Numbers.....	63
Chapter 7 - Imaginary Number Lines	69
Chapter 8 - Place Value	73
Chapter 9 - Alternative Strategies	79
Chapter 10 - One Hundred	85
Chapter 11 - Formal Subtraction: An Option.....	89

Introduction

Grow Your Brain focuses on addition facts and a child's ability to add and subtract. It uses this crucial topic of early math education, one that can easily introduce mathematics to children as essentially a tedious memorization process, as an opportunity to develop number-sense and communicate an understanding of mathematics as a thinking activity.

Dialogs

Grow Your Brain provides resources and strategies for short sessions devoted mostly to an oral dialogue between teacher (whether classroom teacher, tutor, or parent helping or home-schooling) and children.

These daily sessions may last anywhere from 15 to 40 minutes during First and Second grades and beyond as needed.

In a kind of Socratic approach, the dialogs often start with a question that the child has no difficulty answering. We then ask a second question that the child can answer only because of the answer given to the initial question. The two questions teach more than a fact: They teach a connection that applies to many similar facts. Also, the child is not told what needs to be learned, but discovers it as a natural consequence of what he or she already knows. Mathematical knowledge emerges and expands as a coherent network of connections in the mind of the child. It comes from within and is not imposed from the outside as something that needs to be memorized and blurted out on demand.

We present mostly the parent/teacher's side of these dialogs. These are not scripts to be followed to the letter, but models that the parent/teacher will adjust, modify, improvise, and expand upon. The parent/teacher asks questions or gives an instruction. Students respond mostly orally, sometimes in writing, or with an activity involving the use of a few simple tools.

The child's answers provide feedback that guides the parent/teacher as he or she asks the next questions, but these are not asked as a test of a child's knowledge. The questions or prompts are learning and reviewing opportunities.

Chapter 1: First Wave

Identifying Numbers on the Line to 10

Parent/teachers can improvise very simple conversations that help children identify any number on the line, show its relative position, and draw the most basic arithmetic conclusions. Initial prompts:

- Identify the benchmarks:

Show me 5, 0, 10.

- Identify one more or one less than the benchmarks:

Show me 6.

How do you know it is 6?

It is 1 more than 5.

Show me 9, 4, 1.

2, 3, 7, 8 are more difficult to immediately identify on the line.

Patently Building Connections

We dialog freely about a number's position in relation to benchmarks 5 and 10. Children take their time to answer:

Show me 7 (6, 9, 8) and mark it with a token.

Is 7 closer to 5 or to 10?

Closer to 5

How far is 7 from 5?

2 steps

How far is 7 from 10?

3 steps

Show me 7 and show me 3 more.

What's $7 + 3$?

10

Show me 4. How far is 4 from 5?

1 step

Show me 1. How far is 1 from 5?

4 steps

Show me 6. How far is 6 from 5?

1 step

How far is 6 from 10?

4 steps

Put your finger on 0.

Move up 3 steps. Where are you?

On 3

Move up 2 steps. Where are you now?

5

Move up 3 steps again. Where are you?

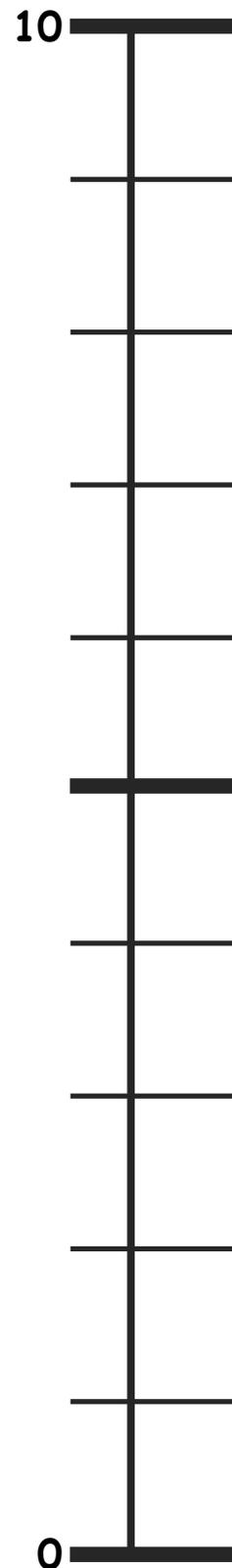
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How far are you from 10?

2 steps

Move up to 10.

Show me again and tell me what you just did.



Changing the Order

$$26 - 9 + 3 - 1 + 4$$

In this sequence, we cannot add $9 + 3$ or $1 + 4$. If we do, we get the wrong answer. But once we draw the bubbles, each number is permanently attached to the operation sign in front of it. Now we can move those bubbles around and group them almost any way we want. For instance, we can change the order to make it easy to group and add numbers in sequence. Children have no difficulty accepting that a bubble with a minus sign cannot be put in an initial position or that a number in the initial position needs a + sign if moved to another position. So we ask:

Draw the bubbles around each number and the sign in front of it.



This bubble says: “Take away 9.” This one says: “Add 3.”

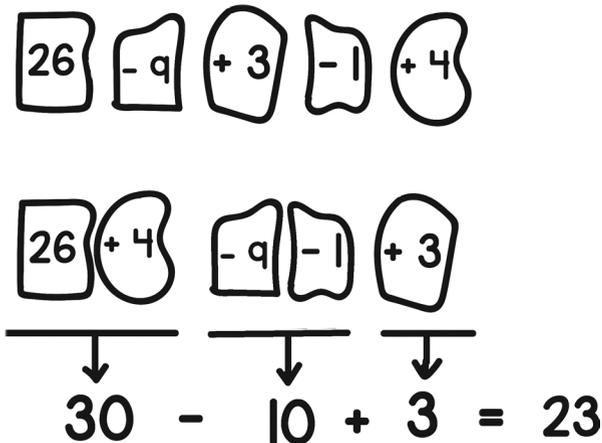
Copy each bubble on separate pieces of paper.

Now each piece of paper is like one of the bubbles.

Move these papers around to group bubbles in ways that make sense to you.

Do you think the order of the bubbles changes the final total?

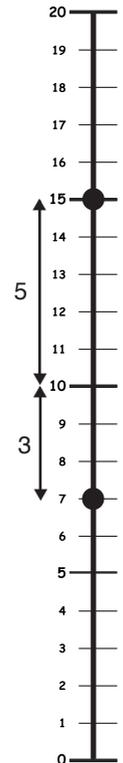
What’s the final amount?



From now on, we can write down a series of operations made up mostly of Make-10 pairs of one kind or another and include that activity in our daily variety show of mathematical activities. Such sequences can also be handed out as short written practice slips.

Number Stories

The use of a vertical number line and benchmark, real initially and then as mental objects, can help children visualize many simple number problems, including some found in other math material used by a class and already solved perhaps by other thinking or drawing strategies. Let's just give a few examples:



**There are 7 frogs in the pond.
8 more jump in to join them.
Show me on the line how you add them.
Starting with 7, think: "3 steps to 10 and 5 more steps to 15."**

**Sylvia goes into a toy store with \$15.00 in her pocket.
She comes out with her favorite toy and only \$7.00.**

Mark those two numbers on the line.

What does 15 represent?

What does 7 represent?

What two numbers do you add to find the difference?

3 and 5

So how much did that toy cost?

\$8.00.

With your finger on the line, show what happens as Sylvia walks into the store with \$15.00 and then comes out with \$7.00.

When Robby goes to bed, there are 15 cookies in the cookie jar.

There are only 7 when he wakes up.

How many did the cookie monster eat while Robby was sleeping?

Mark the two numbers on the line and let's talk.

With the following example, keeping track of the difference in the scores of a basketball game expands the child's use of benchmarks to multiples of 10. The child can explain her reasoning process: (23–17? I use 20 as a benchmark. 3 steps to 20 and 3 more steps to 23 for a difference of 6 points....52–49? One step to 50 and 2 more for a difference of 3 points.)

You're watching a basketball game.

You want to know the difference in the scores.

Tell me the difference as I write down the scores:

With 50 as the benchmark, you think: $1 + 6 = 7$

Team A:	12	12	17	23	25	31	37	44	52	56
Team B:	5	8	9	17	17	22	27	38	47	49

Difference:	7	4	8	6	8	9	10	6	5	7
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Digit in Motion

We draw a series of zeros: 000,000. We prepare a strip of paper that can hide all these zeros, with a single digit, the number 2 for instance, on the right edge of the piece of paper. We let the child read the numbers as we pull the strip to the left to show 2, then 20, next 200, and finally 2,000.

Children experience that the number 2 changes *value* as it changes *place* in a dynamic way that would not be as obvious if they only compared 2 in different positions in static numbers or by just adding zeros.

With the strip keeping up with the questions, we may ask:

Which is more: 2 dollars or 20 dollars?

20 dollars or 200 dollars?

200 or 2,000?

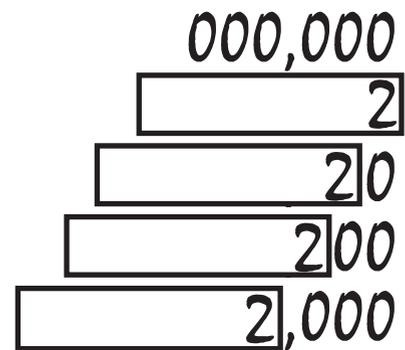
Which is less: 2,000 or 200?

200 or 20?

We've just put the digit 2 in different places.

Does it have the same value as it changes place?

How can we give it a larger value? A smaller one?



Ten Times Larger, Ten Times Smaller

Now you do it.

Hide those zeros and start with 2.

Make that 2 *ten times larger*...

.....

...Now, make 2,000 *ten times smaller*...

With no need for explanations, children are experiencing that the ratio from one place value to the next is 10. We can help them transfer that understanding to the more usual static numbers. We are using the strip. Children write down or cross out zeros.

I hide these zeros to show a 2. Write down 2.

2

Now I pull the paper and change my 2 into 20.

How can you make your 2 ten times larger?

I write a zero to show 20.

Later, at any time, we can ask children to make a number ten times larger or smaller as a 30-second review:

What's 5 + 3 ? Make that 8 ten times larger.....

80

.....Make that 8,000 ten times smaller.

800~~0~~