Fred's Home Companion
Advanced Algebra

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A Note to Students

When you turn to Lesson One in this book, you will find that it asks you to read three pages in *Life of Fred: Advanced Algebra*. Reading a little bit about Fred and his adventures is always a fun way to begin a day. In the first lesson you'll be reading about Fred heading home on the bus. He's wearing his hospital shirt with little blue and green frogs on it that he received in *Life of Fred: Beginning Algebra.*

After you have read those three pages, turn back to this book and answer the questions of Lesson One. All the answers are given on the next page, so you'll know you are on the right track.

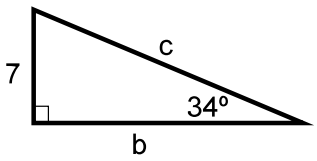
That's it.

Ahead of you are

Life of Fred: Geometry
Life of Fred: Trig
Life of Fred: Calculus.

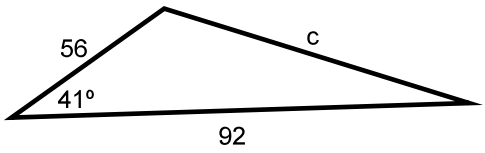
Life of Fred: Geometry is really a book about reasoning. The circles (and triangles and squares and lines and points and rectangles and planes and parallelograms and polygons and angles) just give us something to reason about. You will learn about what makes a valid argument. This is the only mathematics course from kindergarten through the second year of college that concentrates on what it means to think logically. You encounter lots of If–Then statements.

Suppose your mother, speaking hyperbolically, tells you, "If you do *that*, I'm going to kill you." And suppose you do *that*. You might not be ready for geometry. Or, on the other hand, geometry may be exactly what you need.

Life of Fred: Trig deals mostly with triangles. You might be given this right triangle 

and asked to find the values of b and c.

Or in trig you might be given this triangle and asked to find c.

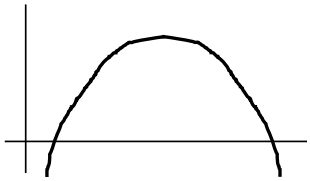


Calculus is the gateway to many different majors in college. All the sciences need it. Even business majors often need it. You may not need calculus if all you are going to take is It Lit.✶

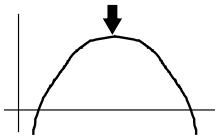
Calculus adds one new concept—that of the idea of limit. The definition of limit is the wellspring from which all of the three or four semesters of calculus flow.

Here are three of the many uses of the idea of limit:

Suppose we have the curve $y = -x^2 + 8x - 12$.

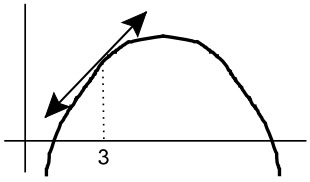


Use #1: We can find the maximum point on the curve.



Use #2: We can find the area under the curve which is above the x-axis.

Use #3: We can find the slope of the line that is tangent to the curve at x = 3.



Now to answer some of the common questions that students have . . .

CAN I USE MY CALCULATOR IN ALGEBRA?

Yes. It is the addition and multiplication tables that you need to know by heart. Once you have them down cold, and you know that the

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✶ "It Lit" is college-talk for Italian literature courses.

area of a triangle is one-half times base times height, there is little else that you should have to sit down and memorize.

When I taught arithmetic, the tests I gave were always taken without the use of a calculator, but when I taught algebra/geometry/ trigonometry/calculus/math for business majors/statistics, the tests were always open-book, open-notes, and use-a-calculator-if-you-want-to.

There are a lot of times in life in which you may need to know your addition and multiplication facts and won't have access to a calculator, but when you are doing algebra or calculus problems you will almost always have a calculator and reference books handy.

WHAT KIND OF CALCULATOR WOULD BE GOOD?

In beginning algebra all you really needed was the basic calculator that has these five keys: $+, -, \times, \div, \sqrt{\ }$.

Now in advanced algebra it is time to buy a "scientific calculator." It will have sin, cos, tan, !, log, and ln keys. The most fun key is the "!" key. If you press 8 and then hit the ! key, it will tell you what $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ is equal to.

Recently, I saw a scientific calculator on sale for less than $8. This will be the last calculator you will need to learn all the stuff through calculus.**

** Some schools require their calculus students to buy a fancy graphing calculator which costs between $80 and $100. I don't own one and I've never needed one. I spent the money I saved on pizza.

A Note to Parents Who Are Homeschooling Their Kids

Fred's Home Companion will put your children on "automatic pilot." Each day they do one (or more) lessons. The reading in *Life of Fred: Advanced Algebra* is fun. And because it is fun, they will learn mathematics much more easily. You can sit back and watch them learn.

Six-year-old Fred first encounters the need for mathematics in his everyday life, and then we do the math. This is true for all the *Life of Fred* books. The math is *relevant*.

In the traditional school settings, all the subjects are packaged into air-tight compartments. The students are like little cars that scurry around during the day. First, they might park themselves in the English "filling station" and get a gallon of English poured into them. At the history filling station they would get history.

For years, educators have bemoaned this compartmentalization. "We are teaching children, not subjects," is a favorite expression of theirs. And yet, things don't seem to change much. If it is nine o'clock and the students are sitting in a French class, they never hear about biology or music. This is not a natural way to learn. Do French people only discuss how irregular their verbs are?

In *Life of Fred: Advanced Algebra* we certainly teach algebra—more than is taught in most schoolrooms—but the whole world is ours to explore and learn about.

In chapter four, when the four-year-old girls eat some doughnut dough, we discuss the **biology**, **chemistry**, and **physics** (p. 117). When they float up in the sky, they seem to take the shape of the constellation Cassiopeia. We do a little **astronomy** and mention how many official constellations there are (p. 124). When the five little girls land in a tree (p. 127), it is time to quote a little **poetry**: ". . . A tree that looks at God all day/And lifts her leafy arms to pray. . . ." which was written by Joyce Kilmer who "died in action on July 30, 1918, near the end of WWI." In the space of two sentences we have mentioned **prayer** in a positive context, **patriotism**, and that World War I ended in 1918 (**history**).

In those same eleven pages (pp. 117–127) we have also done a lot of algebra:

❀ What a line with slope zero looks like,
❀ What a line with a negative slope looks like,
❀ A whole-page argument as to why we use subscripts,
❀ The Greek letters sigma and pi,
❀ The derivation of the formula for the slope of line passing through the points (x_1, y_1) and (x_2, y_2),
❀ The derivation of the distance formula between the points (x_1, y_1) and (x_2, y_2),
❀ The slope-intercept and the double-intercept forms of the line,
❀ The point-slope form of the line,
❀ The concepts of independent and dependent variables, and
❀ The derivation of the two-point form of the line.

A lot happens in *Life of Fred: Advanced Algebra*—a tale of Fred's two-day bus trip from Texas to Kansas.

Contents

Lesson One

Ratios, Median Averages, Proportions

Life of Fred:
Advanced Algebra
pp. 16–18

1. Which is larger: 6:5 or 9:8?
2. In some of the old math books they used to write a proportion as 2:3::6:9. What would the double colon in the middle represent?
3. When Fred first counted the ratio of passing telephone poles to his heartbeats, he found it was 5:3. Suppose the driver of the bus increased his speed. What might the new ratio look like?
4. As Fred was counting the ratio of passing telephone poles to his heartbeats, suppose (Heaven forbid!) his heart stopped beating.

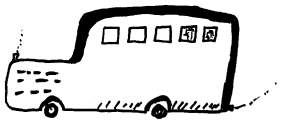
✓ The bus driver wouldn't like this because he would have to stop the bus and do some heart surgery or something.

✓ The readers of the advanced algebra book wouldn't like it because the book would end too soon.

✓ Mathematicians wouldn't like it because the resulting ratio is 5:0. Why would they object?

5. Solve $\frac{x + 3}{x + 13} = \frac{3}{5}$

6. The bus driver is 25 years old. The bus is 35 years old. How long will it be before the driver is 75% of the age of the bus?



old bus

7. What is the median average of:

5, 8, 9, 9, 10, 14, 18, 19, 19?

answers

1. 6:5 means 6 ÷ 5 which is 1.2.
   9:8 means 9 ÷ 8 which is 1.125. 6:5 is larger.
2. A proportion is the equality of two ratios.
   The expression 2:3::6:9 would translate into 2:3 = 6:9 or $\frac{2}{3} = \frac{6}{9}$
3. Instead of 5:3 it might be 6:3 or 7:3. Any answer you gave which was in the form x:3 where x > 5 would have been fine.
4. A ratio of 5:0 means $\frac{5}{0}$ which is division by zero. Mathematicians don't especially like that. It is similar to going up to someone and saying, "The snamplefork is overzipped." Division by zero *doesn't have any meaning.* When you divide 2 into 6 you get an answer of 3.

$$2\overline{)\,6}\;\;3$$

You check your answer by multiplying 2 by 3 and hoping to get 6.

If you try to divide by zero, $0\overline{)\,6}\;\;?$ what could the answer be? What number could you replace the question mark with so that the answer would check? Suppose the answer were 97426398799426.

Suppose $0\overline{)\,6}\;\;97426398799426$

This answer wouldn't check since $0 \times 97426398799426 \neq 6$.

5. $$\frac{x + 3}{x + 13} = \frac{3}{5}$$

$$\frac{(x + 3)\mathbf{5(x + 13)}}{x + 13} = \frac{3\cdot\mathbf{5(x + 13)}}{5}$$ Multiplying both sides by **5(x + 13)**

$$(x + 3)5 = 3(x + 13)$$

$$x = 12$$

6. Let x = the years until the bus driver is 75% of the age of the bus.
   In x years, the bus driver will be 25 + x years old.
   In x years, the bus will be 35 + x years old.
   75% = $\frac{3}{4}$

$$\frac{25 + x}{35 + x} = \frac{3}{4}$$

$$x = 5 \text{ years}$$

7. The median average of 5, 8, 9, 9, 10, 14, 18, 19, 19 is the number in the middle when they are all arranged in order of size. In this case it is 10.

Lesson Two

Solving Proportions by Cross-Multiplying

Life of Fred:
Advanced Algebra
pp. 19–21

1. Solve by cross-multiplying:

$$\frac{x-4}{x+2} = \frac{1}{3}$$

2. Cheryl Mittens is twice as old as the watch she wears. Six years ago (when Fred was born) she was three times older than her watch. How old is her watch now?

Cheryl's watch that she got when she was in middle school

3. Solve $\frac{x-4}{2} = \frac{5}{x+5}$ (Use factoring to solve the resulting quadratic equation.)

4. Solve $\frac{x}{x^2-10} = \frac{5}{3}$ (Use the quadratic formula.)

5. Cross-multiplying works fine with proportions.
$\frac{5}{\pi} = \frac{x}{11}$ turns into $55 = \pi x$.

But when it's not a proportion, we use other approaches.
What would be the *next step* in solving $\frac{5}{\pi} = \frac{x}{11} + \frac{x-3}{x}$

Index

a little note about

Four Algebras

Beginning Algebra
Advanced Algebra
Linear Algebra
Abstract Algebra

Beginning Algebra and Advanced Algebra are the two years of high school algebra. You have just finished all of your high school algebra.

After Geometry and Trig, you will have finished all your high school mathematics and will be ready for the first college mathematics course, which is Calculus. Calculus is a big subject—four semesters of college—and is usually done in the freshman and sophomore years.

When you become a junior in college, you are asked to declare a major (a field of study that you would like to concentrate on). Here are some of your choices:

☒ M a t h e m a t i c s yes! yes! yes!
□Fashion Design
□Fashion Merchandising
□Feed Science
□Fiber, Textiles, and Weaving Arts
□Film
□Finance
□Floriculture
□Food Science
□Forensic Science
□Forestry
□French
□Furniture Design

After you have selected Mathematics as your major (this is merely a suggestion), you and other math majors will take upper-division (junior and senior level) math courses.

If, on the other hand, you select a Film major, you will see a lot of movies. If you are a Feed Science major, you may feed a lot of chickens. If you choose Furniture Design, you may get to go into a workshop and build a chair with six legs. If you elect a triple major, you could end up sitting on a silly chair, watching a movie, and feeding movie popcorn to the chicken who is sitting next to you.

As a math major at the junior level, one of your first courses after Calculus is Linear Algebra. In chapter five of *Life of Fred: Advanced Algebra* we looked at systems of linear equations such as

$$\begin{cases} 7x + 3y + 11z = 8 \\ 5x + 6y - 33z = 9 \\ 9x - 2y + 44z = 7 \end{cases}$$

and solved them by elimination, substitution, and graphing.

In Linear Algebra we look at a zillion other ways to solve systems of linear equations and look at the theory behind that (vector spaces, linear transformations, linear functionals, etc.).

Also as a junior in college you can take your fourth algebra course: Abstract Algebra. In *Life of Fred: Beginning Algebra* we looked at the commutative law of multiplication, $ab = ba$, and the distributive law $a(b + c) = ab + ac$. In Abstract Algebra we take those ideas and build structures using words like *commutative, distributive, associative, identity,* and *inverse.*

Those abstract algebra structures have funny names like groupoids, abelian groups, rings, and fields.

Abstract Algebra is sometimes called Modern Algebra.

Or you can feed chickens. It's your choice.