

Fred's Home Companion Trigonometry

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A Note to Students

When you turn to Lesson One in this book, you will find that it asks you to read five pages in *Life of Fred: Trigonometry*. Reading a little bit about Fred and his adventures is always a fun way to begin a day. In the first lesson you'll be reading about six-year-old Fred being driven home in a limo that George had ordered for him at the end of *Life of Fred: Advanced Algebra*. On the second page you find the description of the "bit of dinner" that the driver/chef prepared for Fred.

After you have read those five pages, turn back to this book and answer the questions in Lesson One. All the answers are given on the next page, so you'll know you are on the right track.

That's it.

Now to answer some of the common questions that trig students have . . .

WHAT KIND OF CALCULATOR WOULD BE GOOD?

It is time to buy a "scientific calculator" if you don't already own one. It will have \sin , \cos , \tan , $!$, \log , and \ln keys. The most fun key is the " $!$ " key. If you press 8 and then hit the $!$ key, it will tell you what $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ is equal to. Recently, I saw one on sale for less than \$8. That's the last calculator you'll need to learn all the stuff through calculus.*

* Some schools require their calculus students to buy a fancy graphing calculator which costs between \$80 and \$100. I don't own one and I've never needed one. I spent the money I saved on pizza.

WHAT IF I START TO FORGET SOME OF MY ALGEBRA BEFORE I GET TO CALCULUS?

Three answers:

First, trigonometry is a short math course. In 94 lessons you will finish all of trig.

Second, the *Life of Fred: Trigonometry* book is sometimes used as a pre-calculus book. It includes five chapters that are labeled “Looking Back” and cover major parts of algebra:

- ✓ functions
- ✓ factoring and fractions
- ✓ graphing
- ✓ inverse functions
- ✓ all the number systems (from the natural numbers
through the complex numbers)

Third, lessons 80–94 of this *Fred’s Home Companion* take you through all the chapters of *Life of Fred: Calculus*, chapter by chapter. A preview of all 24 chapters. We will work on all the things that you will need to remember from arithmetic/algebra/geometry/trig for each chapter of calculus.

For example, chapter four in calculus deals with slope. Lesson 82 in this *Fred’s Home Companion* is a final look at the material you learned in algebra and trig about slope.

Our goal is to make you very ready for calculus.

A Note to Parents

Who Are Homeschooling Their Kids

Fred's Home Companion will put your children on automatic pilot. Each day they do one (or more) lessons. The reading in *Life of Fred: Trig* is fun. And because it is fun, they will learn mathematics much more easily. You can sit back and watch them learn.

Six-year-old Fred first encounters the need for mathematics in his everyday life, and then we do the math. This is true for all the *Life of Fred* books. The math is *relevant*.

We will see a tiny slice of Fred's life in *LOF: Trig*—from Tuesday evening to Wednesday night—but in those 29 hours Fred does a lot of living. And he does more than just trigonometry problems.

There is a natural way to learn—and an unnatural way. Sticking a large group of kids in a sit-up-straight-and-be-quiet classroom, giving them a dose of English for an hour, then herding them to a math classroom for a dose of math, is asking for trouble.

English teachers teach English. History teachers teach history. Auto shop teachers teach auto shop. But who teaches the kids?

Children (and adults!) love to learn. Watch a bunch of eight-year-olds during the summer playing in the back yard. They find bugs (biology). They dig holes (civil engineering). They wonder why the sun doesn't burn up (nuclear physics). They make mud pies (culinary arts).

One subject tumbles into another. And it is fun.

Life of Fred: Trig aims toward that ideal.

★ The topic of continuous and discrete variables takes us into a half-page discussion of one of the plays of Shakespeare (p. 37).

★ When Fred is waiting to see the nurse, he imagines that she will be “a cheerful heir to the legacy of Florence Nightingale.” We outline why she played such a pivotal role in the history of women working outside the home (pp. 177–178).

★ Healthy living is mentioned. *Exercise*: Fred starts his Wednesday morning with “his morning jog around the campus” with the result that “Everything felt so wonderful. He was happy to be alive. . . .” *Diet*: Fred had spent the six years of his life living off vending machine food and pizza with his friends. He has drunk a lot of Sluice during those

years—a soda with a lot of sugar in it. He is 36 inches tall and weighs 37 pounds. In the opening two pages of chapter eight, nurse Florrie introduces Fred to a drink that he’s never had before. Fred really liked it and exclaimed, “This is great. It quenches your thirst and doesn’t leave a nasty aftertaste.” And Florrie added, “And no beer gut either.” The drink is . . . water. *Dental hygiene*: Each night (p. 29 and p. 262) Fred flosses and brushes his teeth. Preaching about these things (“You should exercise more! You should watch your diet! You should floss!”) never seems to work well. But the subliminal message given by Fred’s example may do the trick.

★ Learning English is at least as important as learning trig. Fred owns a llama, which he received at his birthday party in chapter six. At the beginning of chapter seven he spots the current issue of a llama magazine. We explain why it is incorrect to say that he was *anxious* to read it.

We spend a half page (p. 184) describing the positive results of reading the great authors.

And we still have plenty of time to cover more trigonometry than is normally presented in a university classroom.

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Lesson Ninety-four

What You'll Need for Calculus

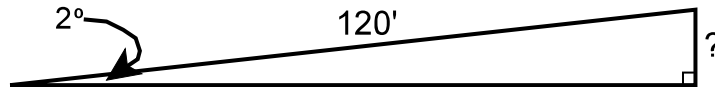
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Lesson One

Angles of Elevation

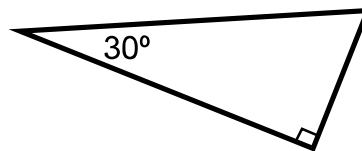
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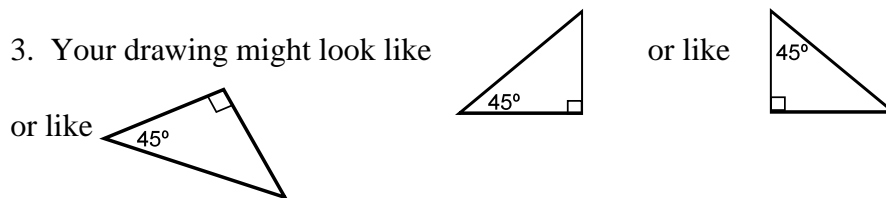
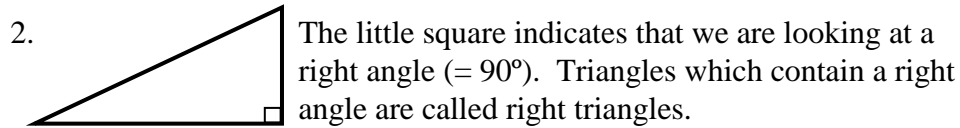
1. In the above triangle the angle of elevation is labeled as 2° . When I measure the angle in my drawing, I find it is actually about 6° . Redraw the triangle more accurately. (Please attempt this first on your own before you look at my answer on the next page.)

The following questions are from geometry.

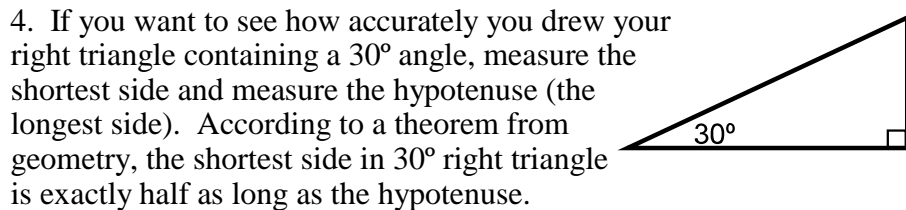
2. What does the little square in the lower-right-hand corner of the above triangle mean?
3. **Acute angles** are angles that are less than 90° . Draw a right angle which has an acute angle of 45° . (You are not required to own a protractor. Just use your ruler and make a rough drawing.)
4. Make a drawing of a right triangle in which one of the acute angles is approximately 30° .
5. If one acute angle in a right triangle is 30° , what is the measure of the other acute angle?
6. A theorem from geometry states, “In any 30–60–90 triangle, the side opposite the 30° angle is half of the length of the hypotenuse.” In the following diagram, mark the side opposite the 30° angle with “opp” and the hypotenuse with “hyp”.



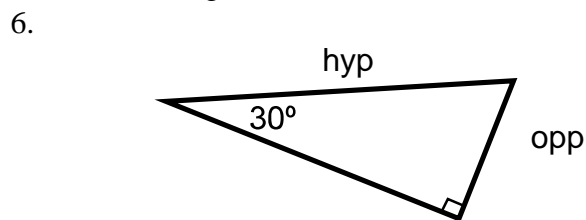
answers



The two shorter sides of the triangle (the legs) you draw should have been roughly equal in length.



5. The sum of all three angles in *any* triangle is equal to 180° . If one angle is 90° and another is 30° , that would leave 60° for the third angle. These right triangles in which one of the acute angles is 30° are sometimes called 30–60–90 triangles.



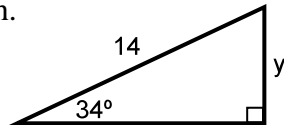
Lesson Two

Definition of the Sine Function

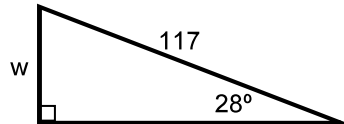
Do the **Y our Turn to Play** starting on p. 22.

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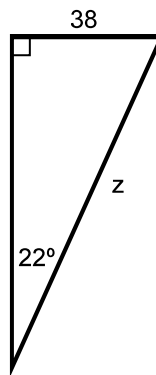
1. Find the value of y . Round your answer to the nearest tenth.



2. Find the value of w . Round your answer to the nearest thousandth.



3. Find the value of z . Round your answer to the nearest hundredth.



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